Homework #1
MATH 7360 – Fall 2021
Due: Friday, Oct 1, 2021

Some R exercises

1. Let $a = 0.7$, $b = 0.2$, and $c = 0.1$.
   - (a) Write out 0.7, 0.2, and 0.1 in binary.
   - (b) In R, test whether $(a + b) + c$ equals 1.
   - (c) In R, test whether $a + (b + c)$ equals 1.
   - (d) In R, test whether $(a + c) + b$ equals 1.
   - (e) Explain what you found. Hint: find out how addition is performed on numerics (double).

2. Create the vector $\boldsymbol{x} = (0.988, 0.989, 0.990, \ldots, 1.010, 1.011, 1.012)$.
   - (a) Plot the polynomial $y = x^7 - 7x^6 + 21x^5 - 35x^4 + 35x^3 - 21x^2 + 7x - 1$ at points x_i in $\boldsymbol{x}$.
   - (b) Plot the polynomial $y = (x - 1)^7$ at points x_i in $\boldsymbol{x}$.
   - (c) Explain what you found.

3. Let $\boldsymbol{u} = (1, 2, 3, 3, 2, 1)^\top$.
   - a Compute $\boldsymbol{U} = \boldsymbol{I} - (2/d)\boldsymbol{u}\boldsymbol{u}^\top$ where $d = \boldsymbol{u}^\top\boldsymbol{u}$. (This type of matrix is known as an 'elementary reflector' or a 'Householder transformation'.)
   - b Let $\boldsymbol{C} = \boldsymbol{U}\boldsymbol{U}$, the matrix product of $\boldsymbol{U}$ and itself. Find the largest and smallest off-diagonal elements of $\boldsymbol{C}$.
   - c Find the largest and smallest diagonal elements of $\boldsymbol{C}$.
   - d Compute $\boldsymbol{U}\boldsymbol{u}$. (matrix times vector).
   - e Compute the scalar $\max_i \sum_j |U(i, j)|$.
   - f Print the third row of $\boldsymbol{U}$.
   - g Print the elements of the second column below the diagonal.
   - h Let $\boldsymbol{A}$ be the first three columns of $\boldsymbol{U}$. Compute $\boldsymbol{P} = \boldsymbol{A}\boldsymbol{A}^\top$.
   - i Show that $\boldsymbol{P}$ is idempotent (in other words $\boldsymbol{P} = \boldsymbol{P}\boldsymbol{P}$) by recomputing (e) with $\boldsymbol{P}\boldsymbol{P} - \boldsymbol{P}$.
   - j Let $\boldsymbol{B}$ be the last three columns of $\boldsymbol{U}$. Compute $\boldsymbol{Q} = \boldsymbol{B}\boldsymbol{B}^\top$.
   - k Show that $\boldsymbol{Q}$ is idempotent by recomputing (e) with $\boldsymbol{Q}\boldsymbol{Q} - \boldsymbol{Q}$.

l Compute $\boldsymbol{P} + \boldsymbol{Q}$.

4. Read in the matrix in the file 'oringp.dat' on the failure of O-rings leading to the Challenger disaster. The columns are flight number, date, number of O-rings, number failed, and temperature at launch. Compute the correlation between number of failures and temperature at launch, deleting the last, missing observation (the disaster).

5. Functions

   a What are the three components of a function?

   b What does the following code return?

```
x <- 10
f1 <- function(x) {
    function() {
        x + 10
    }
}
f1(1)()
```

   c How could you make this call easier to read?

```
mean(, TRUE, x = c(1:10, NA))
```

   d Does the following function throw an error when called? Why/why not?

```
f2 <- function(a, b) {
  return(a * 10)
}
f2(10, stop("This is an error!"))
```

6. Let the $n \times n$ matrix $\boldsymbol{A}$ have elements $A(i, j) = 1/(|i - j| + 1)$.

   a Create a function that takes input argument n and output matrix $\boldsymbol{A}$.

   b Compute and print $\boldsymbol{A}$ for $n = 10$.

   c Compute and print the Cholesky factorization for $\boldsymbol{A}$ for $n = 10$. Hint: try chol() function.

   d Find the determinant of $\boldsymbol{A}$.